



**Birches First School**

*Believe, Grow, Succeed...*

*. to be the best 'me' I can be!*



## Approaches to Maths

### including School Calculations Policy



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# Progression of Methods and Coverage

	EYFS/ Year 1	Year 2	Year 3	Year 4	Year 5
Addition	<ul style="list-style-type: none"> <li>Combining two parts to make a whole: part whole model</li> <li>Starting at the bigger number and counting on – using cubes</li> <li>Regrouping to make 10 using tens frames</li> </ul>	<ul style="list-style-type: none"> <li>Adding 3 single digits</li> <li>Use of base 10 to combine two numbers</li> </ul>	<ul style="list-style-type: none"> <li>Column method – regrouping</li> <li>Using place value counters (up to 3 digits)</li> </ul>	<ul style="list-style-type: none"> <li>Column method – regrouping (up to 4 digits)</li> </ul>	<ul style="list-style-type: none"> <li>Column method – regrouping</li> <li>Use of place value counters for adding decimals</li> </ul>
Subtraction	<ul style="list-style-type: none"> <li>Taking away ones</li> <li>Counting back</li> <li>Find the difference</li> <li>Part-whole model</li> <li>Make 10 using the tens frame</li> </ul>	<ul style="list-style-type: none"> <li>Counting back</li> <li>Find the difference</li> <li>Part-whole model</li> <li>Make 10</li> <li>Use of base 10</li> </ul>	<ul style="list-style-type: none"> <li>Column method with regrouping (up to 3 digits using place value counters)</li> </ul>	<ul style="list-style-type: none"> <li>Column method with regrouping (up to 4 digits)</li> </ul>	<ul style="list-style-type: none"> <li>Column method with regrouping</li> <li>Abstract for whole numbers</li> <li>Start with place value counters for decimals – with the same amount of decimal places</li> </ul>
Multiplication	<ul style="list-style-type: none"> <li>Recognising and making equal groups</li> <li>Doubling</li> <li>Counting in multiples</li> <li>Use cubes, Numicon and other objects in the classroom</li> </ul>	<ul style="list-style-type: none"> <li>Arrays – showing commutative multiplication</li> </ul>	<ul style="list-style-type: none"> <li>Arrays</li> <li>2-digit by 1 digit using base 10</li> </ul>	<ul style="list-style-type: none"> <li>Column multiplication – introduced with place value counters (2 and 3 digit multiplied by 1 digit)</li> </ul>	<ul style="list-style-type: none"> <li>Column multiplication</li> <li>Abstract methods (multi-digit up to 4 digits by a 2-digit number)</li> </ul>
Division	<ul style="list-style-type: none"> <li>Sharing objects into groups</li> <li>Division as grouping e.g. I have 12 sweets and put them into groups of 3, how many groups?</li> <li>Use cubs and draw round 3 cubes at a time</li> </ul>	<ul style="list-style-type: none"> <li>Division as grouping</li> <li>Division within arrays – linking to multiplication</li> <li>Repeated subtraction</li> </ul>	<ul style="list-style-type: none"> <li>Division with a remainder – using lollypop sticks, times table facts and repeated addition</li> <li>2-digit divided by 1 digit using base 10 or place value counters</li> </ul>	<ul style="list-style-type: none"> <li>Division with a remainder</li> <li>Short division (up to 3 digits by 1 digit – concrete and pictorial)</li> </ul>	<ul style="list-style-type: none"> <li>Short division (up to 4 digit numbers by a 1 digit number including remainders)</li> </ul>

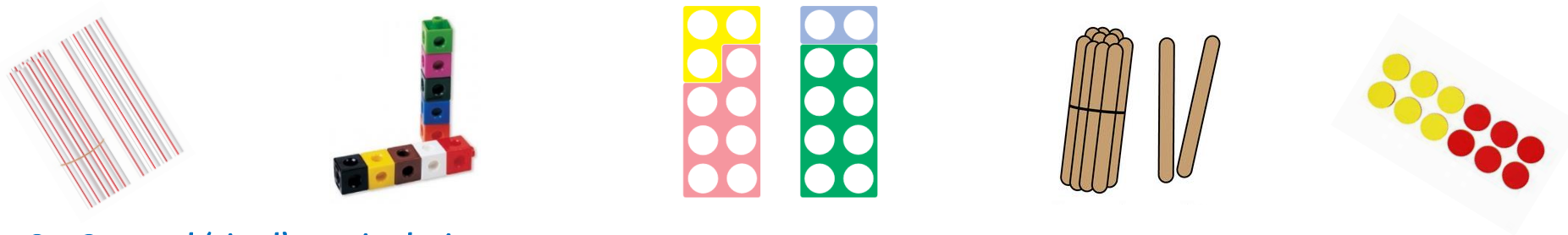
<b>Addition</b>	Practical combining of 2 groups – <b>counting all</b>	Practical combining of 2 groups – <b>counting on</b>	Regrouping to make 10 (numbers to 20)	Adding TO + T or O no regrouping <b>expanded</b>	Adding TO + T or O no regrouping <b>compact</b>	Adding TO + T or O with regrouping (ext to TO+TO and HTO+TO) <b>expanded</b>	Adding TO + T or O with regrouping(ext to TO+TO and HTO+TO) <b>compact</b>
<b>Subtraction</b>	Practical taking away of objects from group	Counting back on a number line	Subtracting TO-T or O no regrouping <b>expanded</b>	Subtracting TO-T or O no regrouping <b>compact</b>	Subtracting TO - T or O with regrouping(ext to TO-TO and HTO-TO) <b>expanded</b>	Subtracting TO - T or O with regrouping(ext to TO-TO and HTO-TO) <b>compact</b>	
		Finding the difference – counting on		Subtraction on a number line – counting back and Finding the difference – counting on TO - T or O (ext to TO-TO and HTO-TO) within 10 ext to bridging 10.			
<b>Multiplication</b>	'Lots of' grouping objects	Repeated addition	Arrays	Grid method	Column method – expanded	Column method – compact	
<b>Division</b>	Sharing into equal groups	Grouping / repeated subtraction on a number line	Grouping / repeated subtraction with remainders	Sharing – with place value counters (early introduction to short division)	Short division		

# Progression of Place Value Manipulatives to Support C/P/A

The initial use of concrete manipulatives is key to developing a good conceptual understanding of place value. Children need lots of opportunities to explore and manipulate a variety of Base Ten materials.

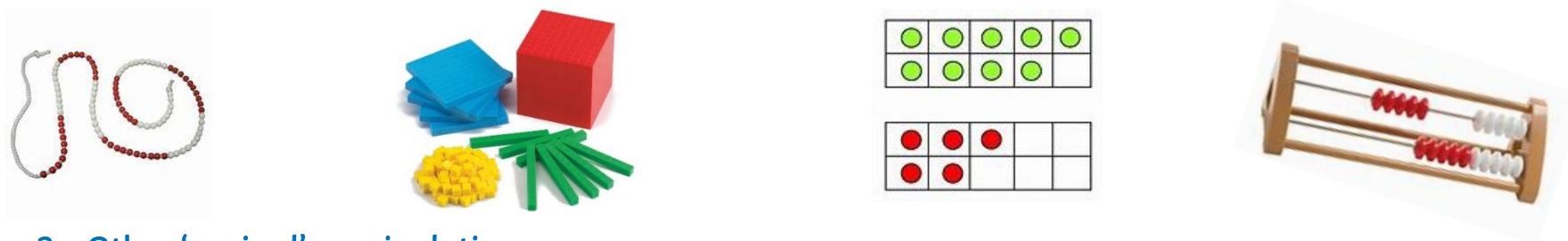
- **Step 1 - Groupable manipulatives**

Manipulatives that can be physically put together in collections of tens and then physically taken apart.  
For example: lollipop/bundling sticks, straws, multilink cubes, counters. Numicon etc.



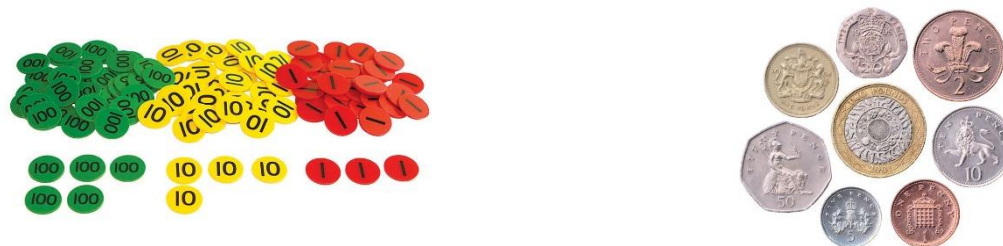
- **Step 2 - Grouped 'sized' manipulatives**

Manipulatives that are already put together as tens, ones etc. The ten is ten times bigger than the one and the hundred is ten times bigger than the ten etc., e.g. Base Ten Blocks/Dienes, ten frames, bead strings, rekenrek etc.



- **Step 3 - Other 'unsized' manipulatives**

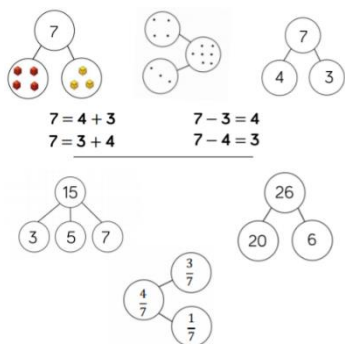
Manipulatives that operate on a base-ten system, whilst not increasing/ decreasing in size by 10 e.g. money, place value counters.



Consideration should be given to the fact that money is included within the KS2 resources for Place Value yet features within the KS1 curriculum.

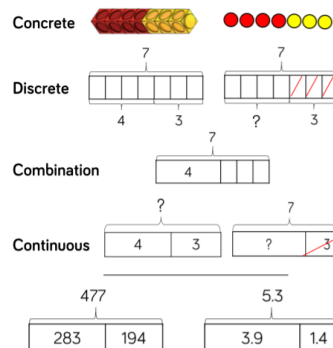
# Models/ Manipulatives to Support the Teaching of Addition and Subtraction

## Part-Whole Model



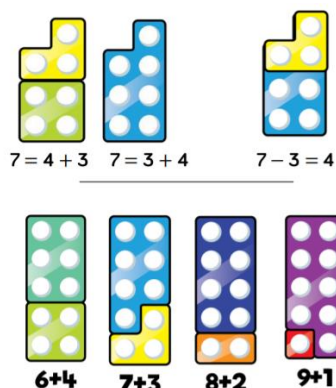
- When the parts are complete and the whole is empty, the children can add the parts together to find the total.
- When the whole is complete and at least one of the parts is empty, children can use partitioning to find the missing part.
- Part-whole models can be used to partition a number into two or more parts or help children to partition a number into tens and ones or other place value columns.
- In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

## Bar Model



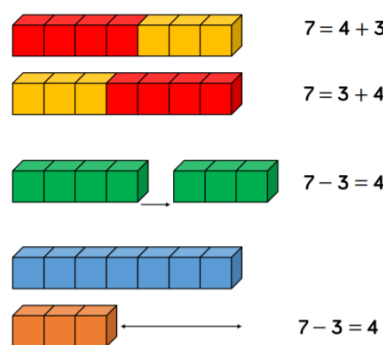
- The bar model is a type of a part-whole model that can support children in representing calculations to help them unpick the structure.
- Cubes and counters can be used in a line as a concrete representation of the bar model. Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.
- The combination bar model can support children to calculate by counting on from the larger number. This can prepare children for the continuous bar model. In a continuous bar model, each rectangle represents a number. The question mark indicates the missing value.
- In KS2, children can use bar models to represent larger numbers, decimals and fractions.

## Numicon



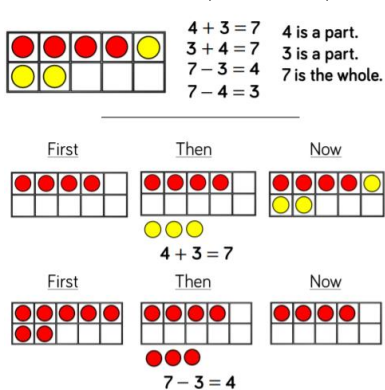
- Numicon can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.
- When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.
- When subtracting numbers, children start with the whole and then place one of the missing parts on top of the whole to see what part is missing. Children will start to subitise the missing part due to their familiarity with the shapes.
- Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all of the possible number bonds for a number.

## Cubes



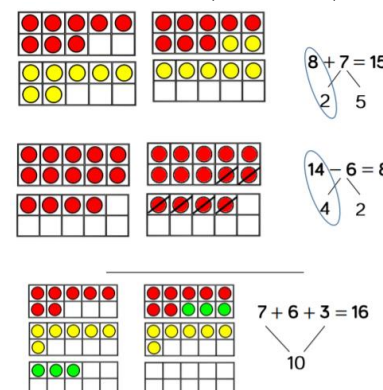
- Cubes can be useful to support children with the addition and subtraction of one-digit numbers.
- When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.
- When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction or take away.
- Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between numbers.
- Cubes are useful when working with smaller numbers but are less efficient with larger number as they are difficult to subitise and children may miscount them.

## Ten Frames (within 10)



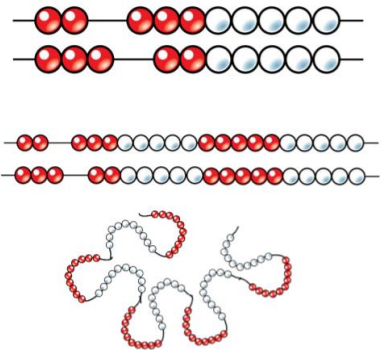
- When adding/subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.
- Using the language of parts and wholes represented by objects on the ten frame introduce children to aggregation and partitioning.
- Using these structures, the ten frame can enable children to find all the number bonds for a number.
- Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. A story structure can help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

## Ten Frames (within 20)



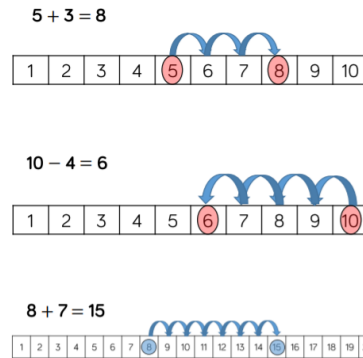
- When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.
- When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.
- When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering the order (looking for number bonds to 10). Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

### Bead Strings



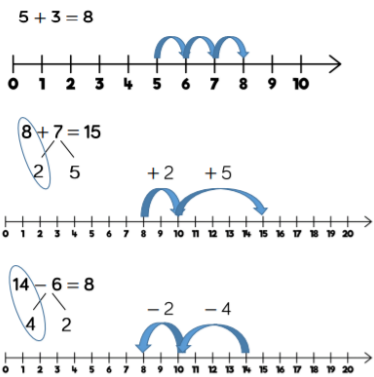
- Different sizes of bead strings can support children at different stages of addition and subtraction.
- Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
- They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different =numbers they have partitioned the 10 beads into e.g.  $2+8=10$ , more one bead,  $3+7=10$ .

### Number Tracks



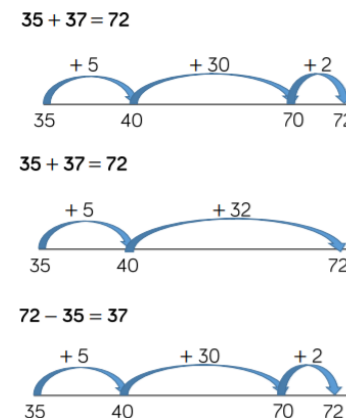
- Number tracks are useful to support children in their understanding of augmentation and reduction.
- When adding, children count on to find the total of the numbers. On the number track, children can place a counter on the starting number and then count on to find the total.
- When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.
- Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.
- Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

### Number Lines (labelled)



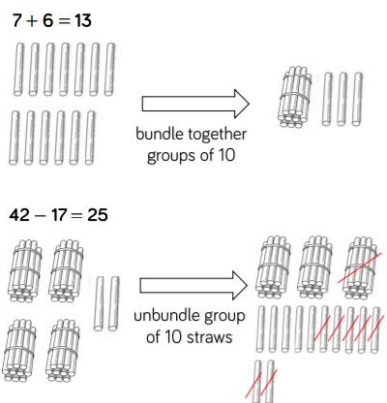
- Labelled number lines can support children in their understanding of addition and subtraction as augmentation and reduction.
- Children can start by counting on or back in one,s up or down the number line. This skill links directly to the use of the number track.
- Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.
- Children can subtract numbers by furstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

### Number Lines (blank)



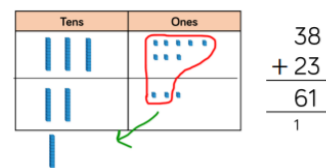
- Blank number lines provide children with a structure to add and subtract numbers in smaller parts.
- Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.
- Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.
- Blank number lines can also be used effectively to help children subtract by find the difference between numbers. This can be done by strating with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

### Straws/ Lollipop Sticks



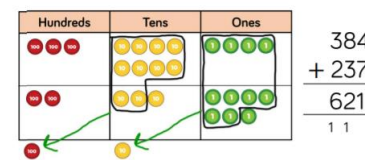
- Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.
- Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numebrs. Use elastic bands or other ties to make bundles of ten straws.
- When adding number, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.
- When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.
- Straws provide a good stepping stone to adding and subtracting with base 10/ dienes.

### Bases 10/ Dienes



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

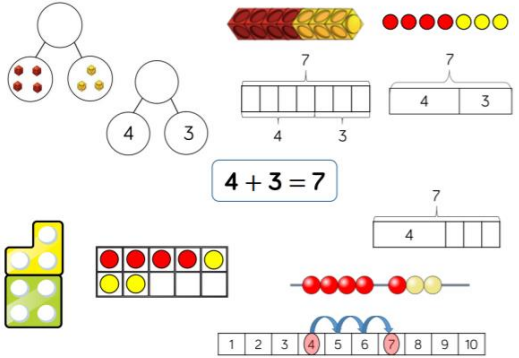
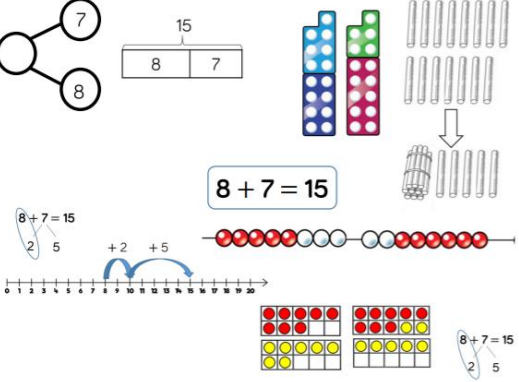
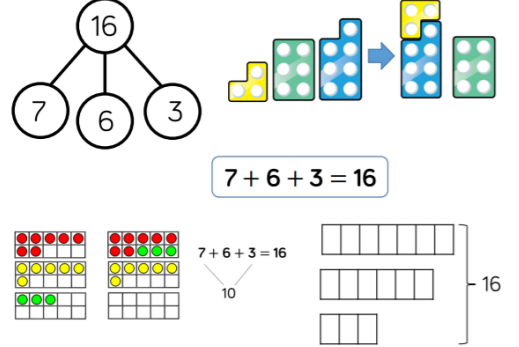
### Place Value Counters

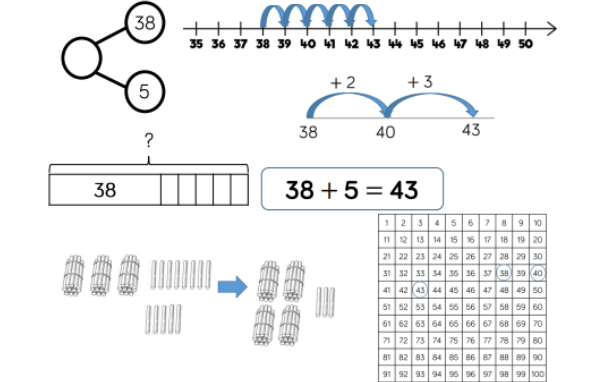
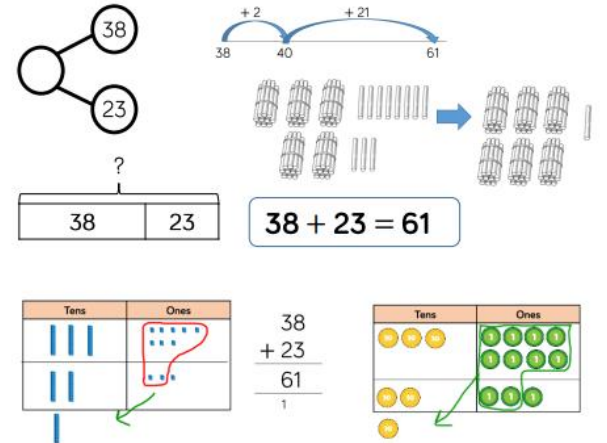
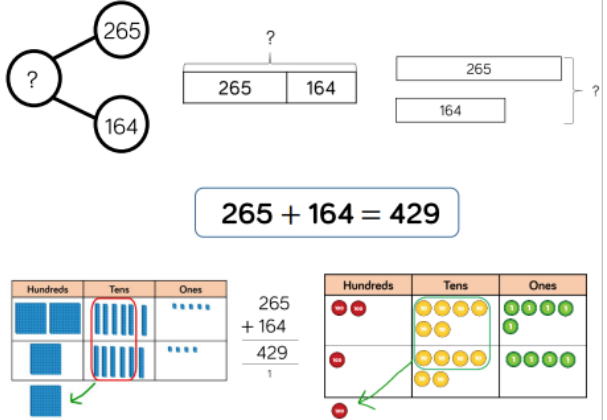


$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 1 \end{array}$$

- Using base 10, dienes and place value counters are an effective way to support children's understanding of column addition and subtraction.
- It is important that children write out their calculations alongside using or drawing base 10/ counters so they can see the clear links between the written method and the model.
- The representation becomes less effiecient with larger numbers when using dienes due to the size of base 10. In this case, place value counters may be the better model to use.
- When adding/ subtracting money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

# Representations and Models – Addition

Typical Year Group	Skill	Representations and Models	
1	Add two 1-digit number to 10	Part-whole model Bar model Numicon	
1	Add 1 and 2-digit numbers to 20	Part-whole model Bar model Numicon Tens frames (within 20)	
2	Add three 1-digit numbers	Part-whole model Bar model	

<p>2/3</p>	<p>Add 1 and 2-digit number to 100</p>	<p>Part-whole model Bar model Number lines (labelled) Place Value Chart</p>	<p>Number lines (blank) Straws Hundred squares</p> 
<p>2/3</p>	<p>Add two 2-digit numbers</p>	<p>Part-whole model Bar model Number lines (blanks) Straws</p>	<p>Place Value Chart Base 10 Place value counters Column addition</p> 
<p>3</p>	<p>Add with up to 3-digits</p>	<p>Part-whole model Bar Model Place Value Chart</p>	<p>Base 10 Place value counters Column addition</p> 

4

Add with up to 4-digits

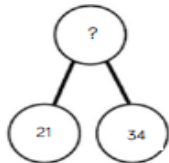
Part-whole model  
Bar Model  
Place Value Chart

Base 10  
Place value counters  
Column addition

$1,378 + 2,148 = 3,526$

Thousands	Hundreds	Tens	Ones
1	3	7	8
2	1	4	8
3	5	2	6
			11

### Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:  
In year 3, there are 21 children and in year 4, there are 34 children.  
How many children in total?

$21 + 34 = 55$ . Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

=  $21 + 34$

Calculate the sum of twenty-one and thirty-four.



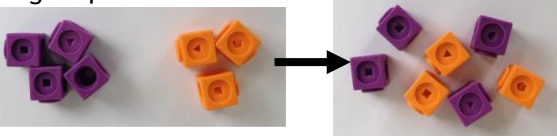
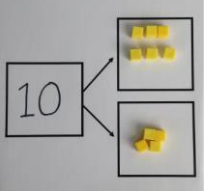

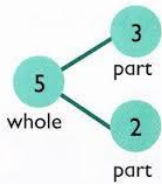

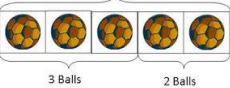

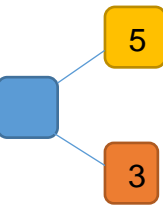

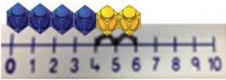

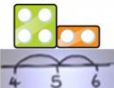
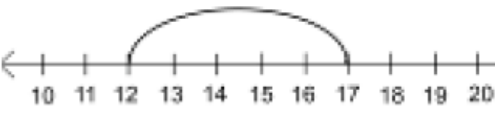
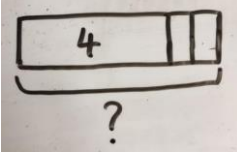
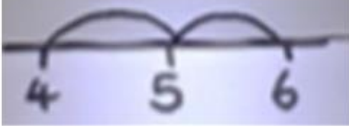
Missing digit problems:

10s	1s
●●	●
●●●	?
?	5



# Progression in calculations: Addition

Key Vocabulary: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as'.

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole</p> <p>Part-whole model</p>	<p>Use cubes to add two numbers together as a group:</p>   <p>Encourage children to move the object as they count so they do not count it twice.</p> <p>Or in a bar:</p> 	<p>Use pictures to add two numbers together as a group:</p>   <p>Or in a bar:</p>   <p>Encourage children to cross out the pictures as they count so they do not count it twice.</p>	<p>Use the part-part whole diagram as shown below to move into the abstract.</p>  <p><math>5 + 3 = 8</math></p> <p><math>10 = 6 + 4</math></p>
<p>Starting at the larger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>    <p>Counting on using number lines, cubes or Numicon.</p>	 <p><math>12 + 5 = 17</math></p> <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>  <p>A bar model which encourages the children to count on, rather than count all.</p>	<p><math>5 + 12 = 17</math></p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p> <p>The abstract number line:</p>  <p>What is 2 more than 4? What is the sum/total of 4 and 2?</p>

**Regrouping to make 10.**

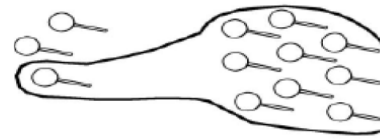


$6 + 5 = 11$



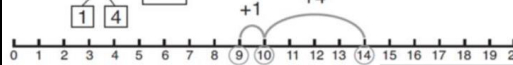
Start with the bigger number and use the smaller number to make 10.

Use pictures or a number line. Regroup or partition the smaller number to make 10.

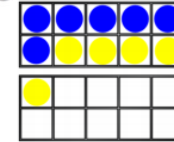


$3 + 9 =$

$9 + 5 = 14$



Children draw the ten frame and counters/cubes



$7 + 4 = 11$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

To develop an understanding on equality:

Children should know that the = sign means 'the same as' / 'equal to'

$6 + \square = 11$

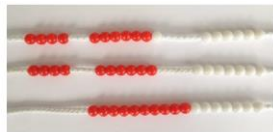
$6 + 5 = 5 + \square$

$6 + 5 = \square + 4$

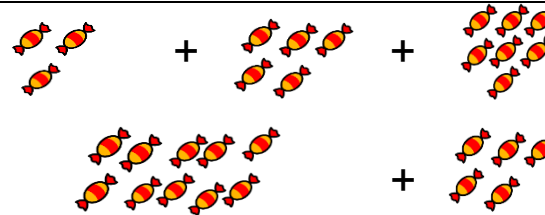
**Adding three single digits**

$4 + 7 + 6 = 17$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

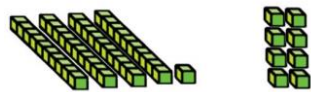
$(4 + 6) + 7 = 10 + 7 = 17$

Combine the two numbers that make 10 and then add on the

remainder.

**Column method- no regrouping (reliant on a secure understanding of place value and partitioning)**

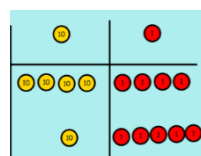
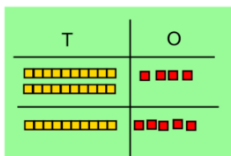
Developing understanding of partitioning and place value;  $41 + 8$



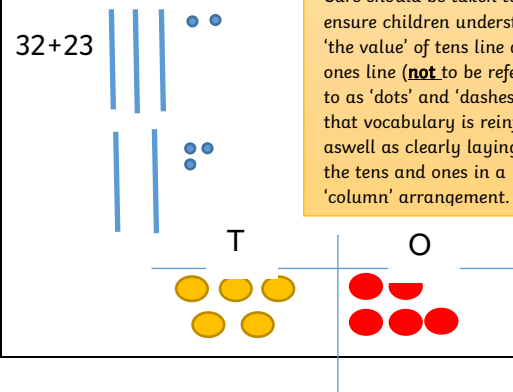
Add together the ones first then add the tens. Use the Base 10 blocks (KS1) before moving onto place value counters (KS2 when secure).

$24 + 15 =$

$44 + 15 =$



After practically using the base 10 blocks (KS1) and place value counters (KS2), children can draw the 'tens and ones'/counters to help them to solve additions.



Care should be taken to ensure children understand 'the value' of tens line and ones line (**not** to be referred to as 'dots' and 'dashes' so that vocabulary is reinforced) as well as clearly laying out the tens and ones in a 'column' arrangement.

Expanded column addition by partitioning first (to secure place value understanding)

$20 + 1$

$40 + 2 +$

$60 + 3$

Moving onto compact method (only when secure) -

Calculations

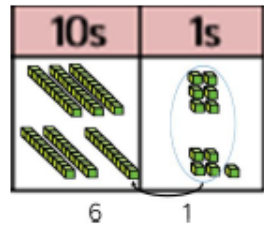
$21 + 42 =$

$21$

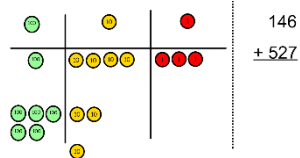
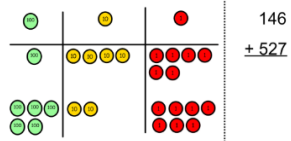
$+ 42$

**Column method-involving regrouping**

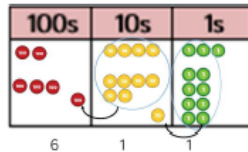
Continue to develop understanding of partitioning and place value.



Practically make both numbers on a place value grid.



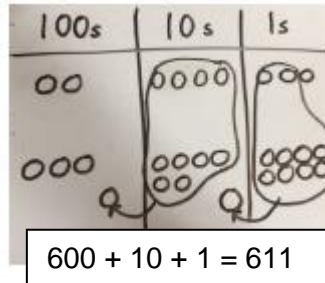
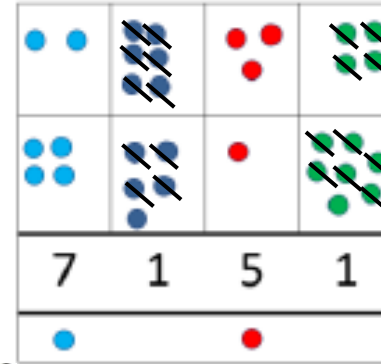
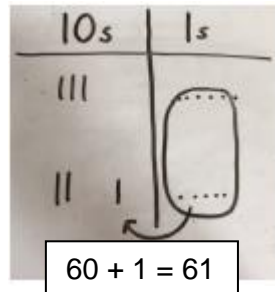
Add up the ones and exchange 10 ones for one ten.



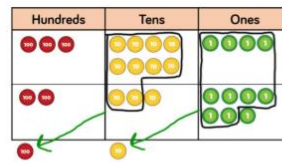
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

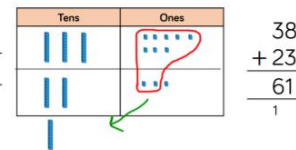
Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



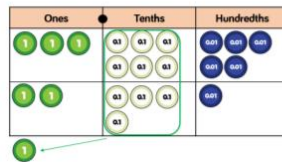
To ensure consistency and avoid confusion, any tens that need to be exchanged should be shown BELOW the answer, not above. The addition symbol should be positioned to the right of the calculation to avoid issues with any place holders.



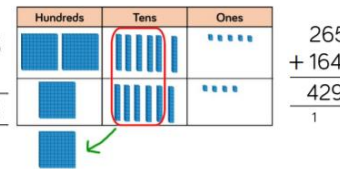
$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 11 \end{array}$$



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$



$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$

Expanded column method first - to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

Moving onto compact column method

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

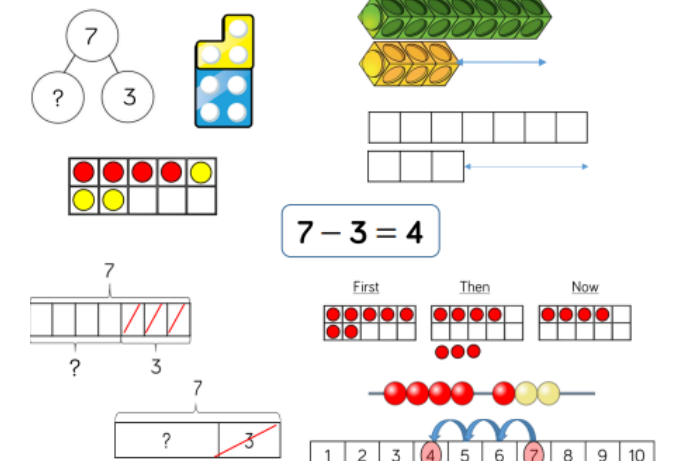
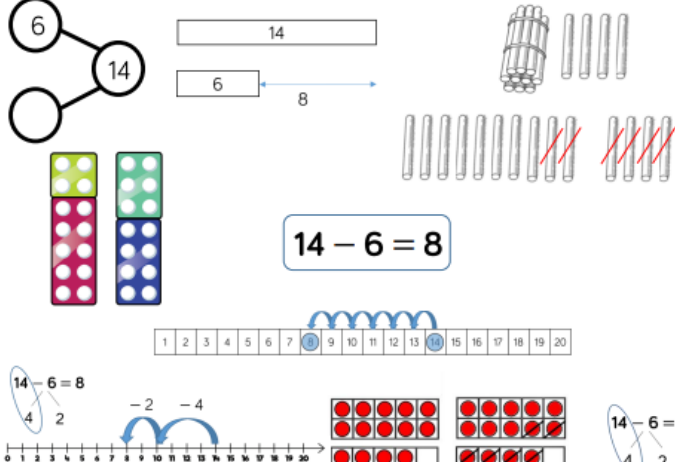
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} £ 23.59 \\ + £ 7.55 \\ \hline £ 31.14 \end{array}$$

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

# Representations and Models – Subtraction

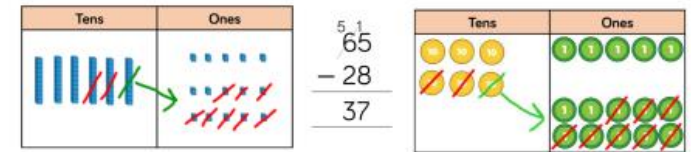
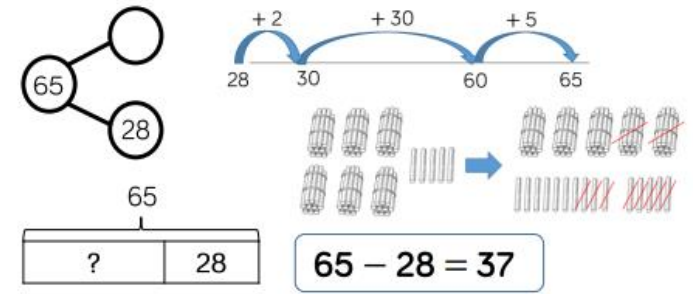
Typical Year Group	Skill	Representations and Models	
1	Subtract two 1-digit number to 10	Part-whole model Bar model Numicon	 <p>7 - 3 = 4</p>
1/2	Subtract 1 and 2-digit numbers to 20	Part-whole model Bar model Numicon Tens frames (within 20)	 <p>14 - 6 = 8</p>

2

Subtract 1 and 2-digit numbers to 100

Part-whole model  
Bar model  
Number lines (labelled)

Number lines (blank)  
Straws  
Hundred squares

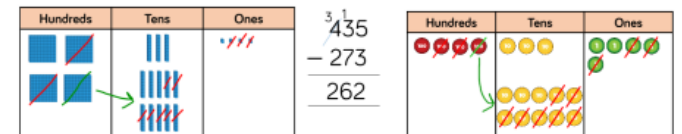
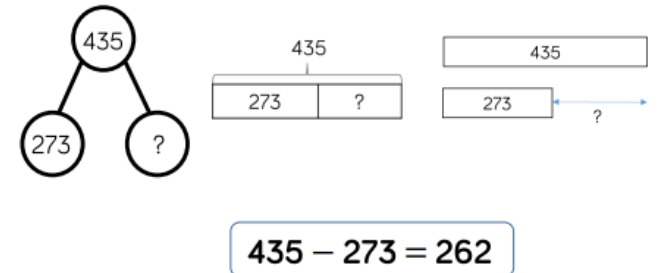


3

Subtract with up to 3-digit digits

Part-whole model  
Bar Model

Base 10  
Place value counters  
Column addition

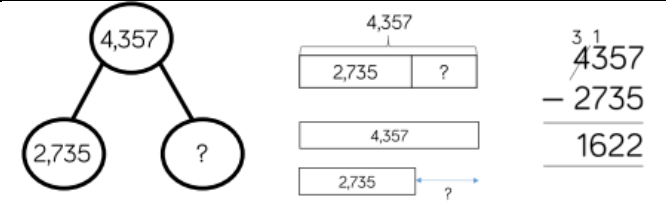


4

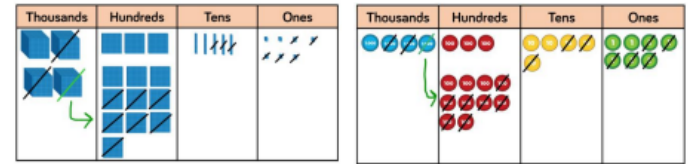
Subtract with up to 4-digits

Part-whole model  
Bar Model

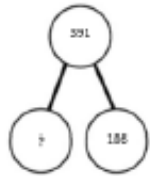
Base 10  
Place value counters  
Column addition



$$4,357 - 2,735 = 1,622$$



### Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.  
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

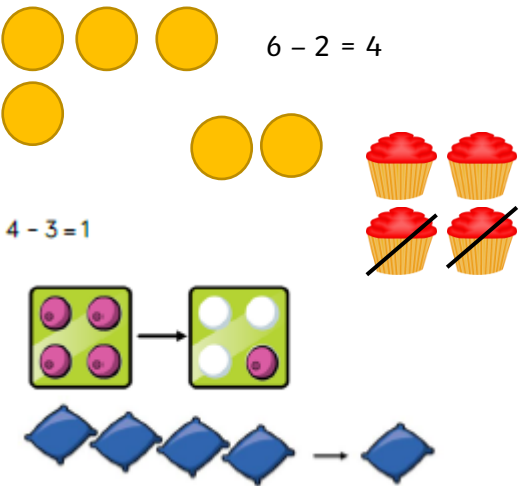
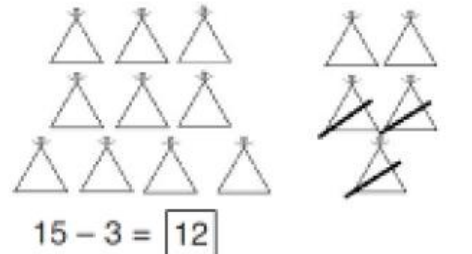
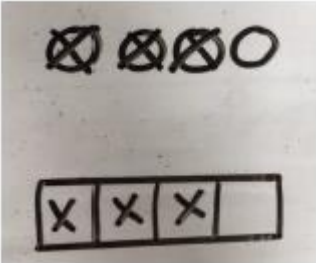
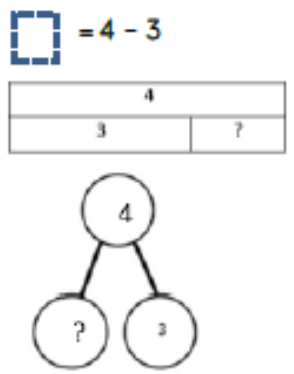

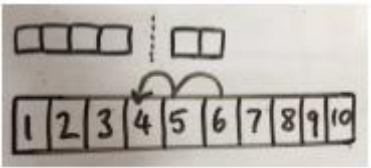
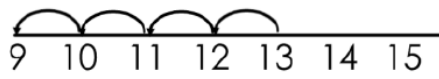
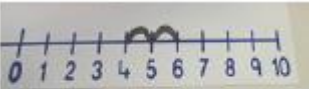
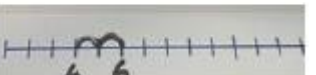
What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ -\square\square6 \\ \hline \square05 \end{array}$$

# Progression in calculations: Subtraction

Key Vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease.

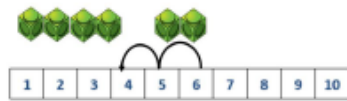
Objective and Strategies	Concrete	Pictorial	Abstract
<p><b>Taking away ones</b></p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away from the whole.</p>  <p><math>6 - 2 = 4</math></p> <p><math>4 - 3 = 1</math></p>	<p>Children draw the concrete resources they are using and cross out the correct amount.</p>  <p><math>15 - 3 = 12</math></p> <p>Bar model can also be used.</p> 	<p><math>18 - 3 = 15</math></p> <p><math>8 - 2 = 6</math></p> <p><math>4 - 3 =</math></p> <p><math>\square = 4 - 3</math></p> 
<p><b>Counting back</b></p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p><math>13 - 4</math></p>	<p>Children represent what they can see</p>  <p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> <p>57-23</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> <p>Children use an empty number line to count back in jumps of H/T/Os depending on level.</p>  

Use counters and move them away from the group as you take them away counting backwards as you go.

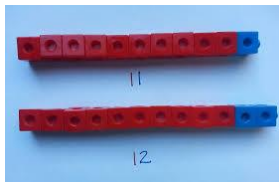


Use practical objects to link to a number track

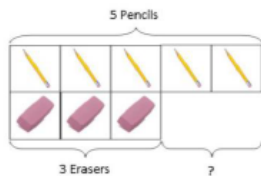
$$6 - 2 = 4$$



Compare amounts and objects to find the difference.

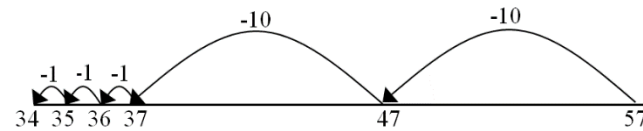


Use cubes to build towers or make bars to find the difference



Use basic bar models with items to find the difference

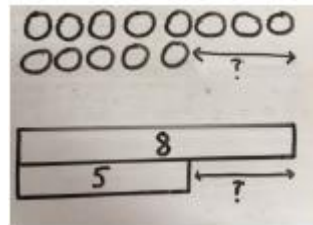
## Find the difference



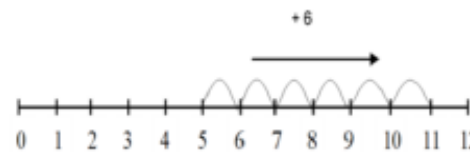
This can progress all the way to counting back using two 2 digit numbers.

Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. How many more/fewer than?

Children explore subtracting on a number line by taking away and finding the difference to secure concept of the inverse.



Children draw objects to support calculation and concept of 'difference'.

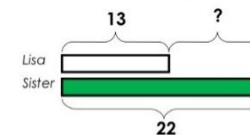


Count on to find the difference.

Draw bars to find the difference between 2 numbers.

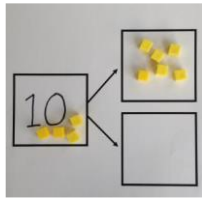
### Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.





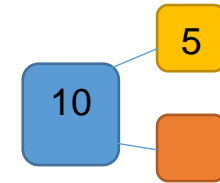
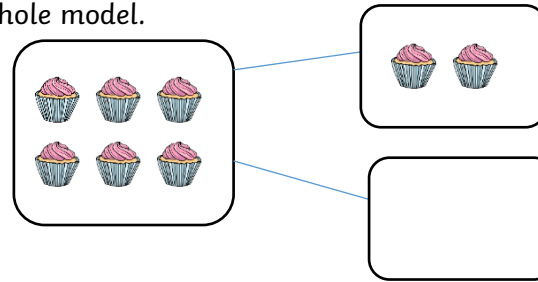
## Part-Whole Model



Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?  
 $10 - 6 =$

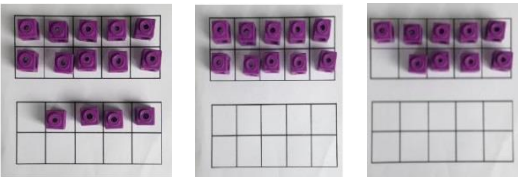
Use a pictorial representation of objects to show the part whole model.



Move to using numbers within the part whole model.

## Make 10

$14 - 5 =$

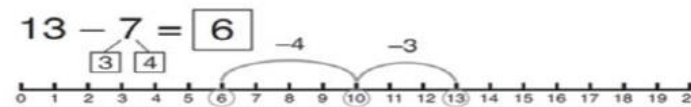


Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.

Children represent the ten frame pictorially



Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.



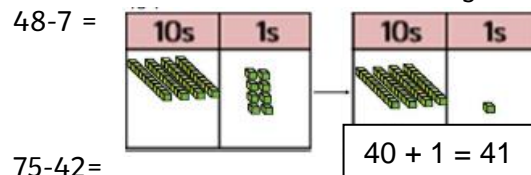
Children show how they make 10 by partitioning the subtrahend (subtracted number)

$$14 - 5 = 9$$

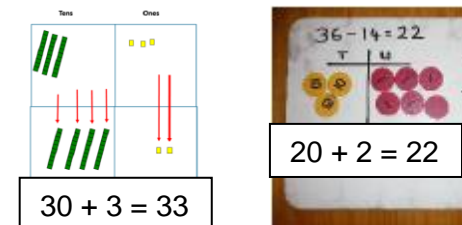
$16 - 8 =$   
 How many do we take off to reach the next 10?  
 How many do we have left to take off?

## Column method without regrouping

Use Base 10 to make the largest number then take the smaller number away.

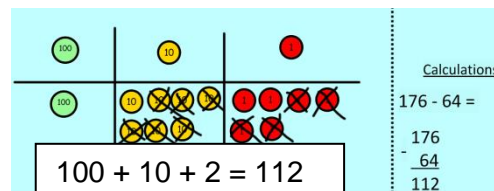
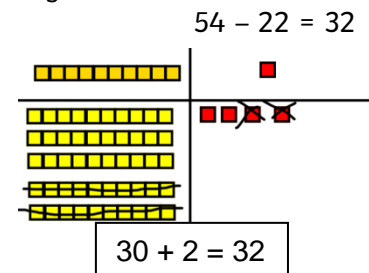
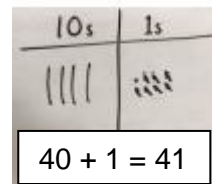


$75 - 42 =$



$30 + 3 = 33$

Represent the Base 10 pictorially



Calculations

$$176 - 64 =$$

$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$

Expanded column method by partitioning first (to secure place value understanding) -

$$\begin{array}{r} 40 \ 8 \\ - 7 \\ \hline 40 \ 1 \end{array}$$

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

Moving onto compact method -

## Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

$10 + 5 = 15$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now I can subtract my ones.

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now I can take away eight tens and complete my subtraction

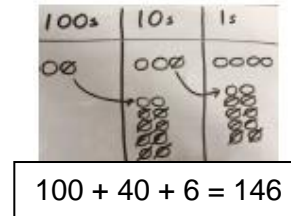
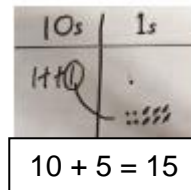
Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$$

Represent Base 10/place value counters pictorially remembering to show the exchange.

$41 - 15$

$234 - 88$



When confident, children can find their own way to record the exchange/regrouping.

NB – whilst neither is incorrect, to ensure consistency and avoid confusion, when teaching column subtraction at Birches First School, any ones or tens that need to be exchanged should be shown above the larger number, not below.

The subtraction symbol should be positioned to the right of the calculation not left to avoid issues with place holders/value.

Expanded column method first - to clearly show the exchange above the subtraction .

$$\begin{array}{r} 41 - 15 \\ \hline 26 \\ \hline \end{array}$$

$$\begin{array}{r} 536 - 85 \\ \hline 451 \\ \hline \end{array}$$

$$\begin{array}{r} 400 & 40 & 9 \\ - & 40 & 16 \\ \hline 360 & 0 & 16 \\ - & 80 & 5 \\ \hline 280 & 0 & 11 \\ - & 20 & 6 \\ \hline 260 & 0 & 5 \end{array}$$

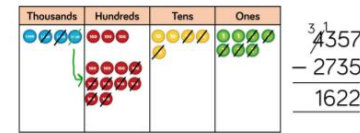
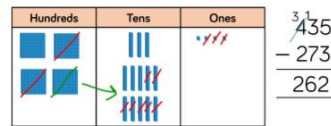
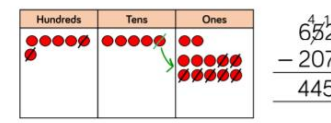
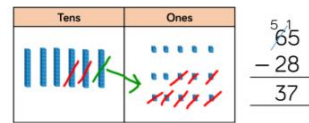
$$\begin{array}{r} 836 - 254 = 582 \\ \hline \end{array}$$

Moving onto compact column method

Children can start their formal written method by partitioning the number into clear place value columns.

$$\begin{array}{r} 728 - 582 = 146 \\ \hline \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

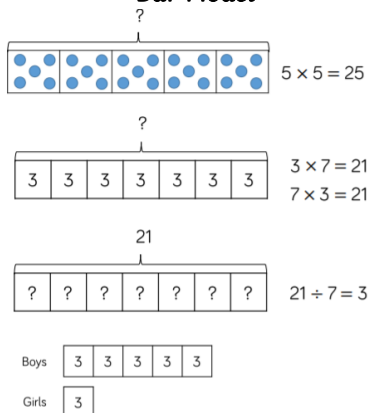


This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

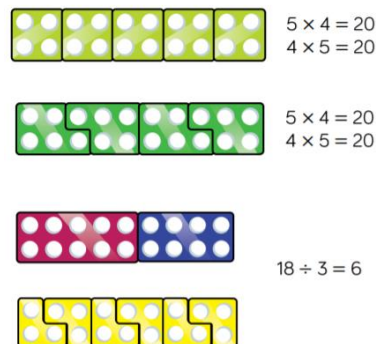
# Models/ Manipulatives to Support the Teaching of Multiplication and Division

## Bar Model



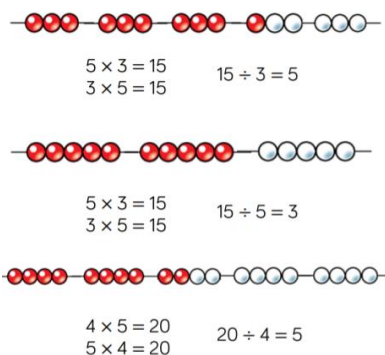
- Children can use a bar model to represent multiplication as repeated addition.
- They can use counter, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent multiplication.
- Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.
- It is important when solving word problems that the bar model represents the problem.
- Sometimes children may look at scaling problems. In this case, more than one bar model is useful to represent the type of problem.

## Numicon



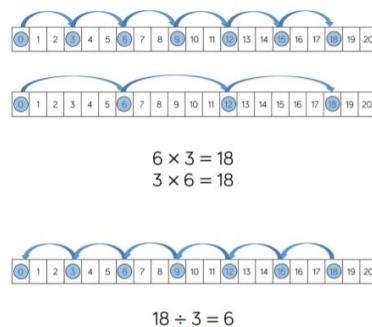
- Numicon supports children's understanding of multiplication as repeated addition.
- Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in a row. Then they can use the tens number shapes along with other necessary shapes over the top of the row to check the total.
- Using Numicon in multiplication can support children in discovering patterns of multiplication e.g. odd  $\times$  odd = even. Odd  $\times$  even = odd. Even  $\times$  even = even
- When dividing, Numicon supports children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether.

## Bead Strings



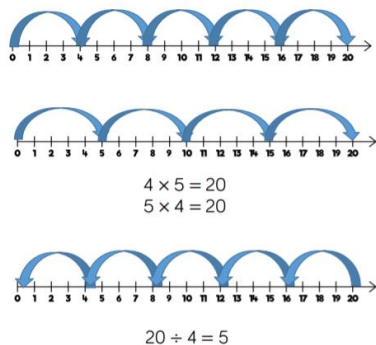
- Bead strings to 100 can support children in their understanding of multiplication and repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
- Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16
- When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4. Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

## Number Tracks



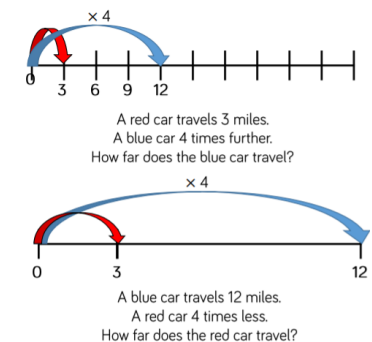
- Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent colours help children to see the number they have landed on whilst counting.
- When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
- When dividing, children place their counter on the number they are dividing and count back in jumps of the number they are dividing by until they reach 0.
- Children record how many jumps they have made to find the answer to the division.
- Number tracks can be useful with smaller multiples but when reaching larger numbers, they can become less efficient.

## Number Lines (labelled)



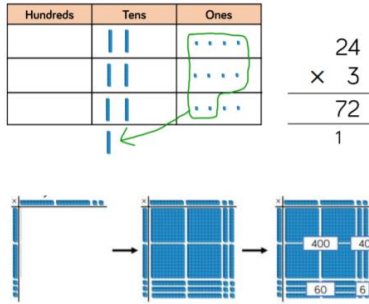
- Number tracks are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.
- When multiplying, children place their counter on 0 and then count on to find the product of the numbers.
- When dividing, children place their counter on the number they are dividing and count back in jumps of the number they are dividing by until they reach 0.
- Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

## Number Lines (blank)



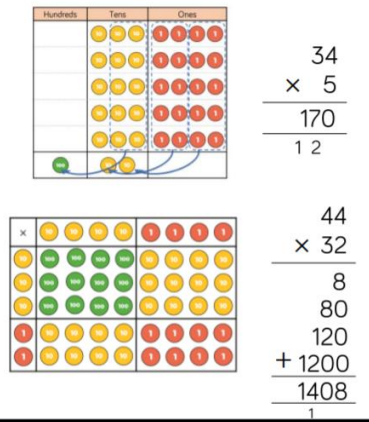
- Children can use blank number lines to represent scaling as multiplication or division.
- Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.
- Blank number lines without intervals can also be used for children to represent scaling.

### Base 10/ Dienes



$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ 1 \phantom{0} \\ \hline \end{array}$$

### Place Value Counters



$$\begin{array}{r} 34 \\ \times 5 \\ \hline 170 \\ 1 \phantom{0} \\ \hline \end{array}$$

$$\begin{array}{r} 44 \\ \times 32 \\ \hline 8 \\ 80 \\ 120 \\ + 1200 \\ \hline 1408 \\ 1 \phantom{0} \\ \hline \end{array}$$

- Using Base 10, Dienes and place value counters are an effective way to support children's understanding of column multiplication and division. It is important that children write out their calculation alongside the equipment so they see how the concrete and written representations match.
- As numbers become larger in multiplication or the amounts of groups become higher, Base 10/Dienes become less efficient due to the amount of equipment and number of exchanges needed.
- When numbers become larger in division, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. in circles or rows on a place value grid.
- Base 10/ place value counters also support the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or formal column method of multiplying 2-digits by 2-digits.
- When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

# Representations and Models – Times Tables

Typical Year Group	Skill	Representations and Models	
2	Recall and use multiplication and division facts for the 2-times table	Bar model Numicon Counters Money	Tens frames Bead strings Number lines Everyday objects
2	Recall and use multiplication and division facts for the 5-times table	Bar model Numicon Counters Money	Tens frames Bead strings Number lines Everyday objects
2	Recall and use multiplication and division facts for the 10-times table	Hundred squares Numicon Counters Money	Bead strings Number lines Base 10 Ten frames
3	Recall and use multiplication and division facts for the 3-times table	Hundred squares Numicon Counters	Bead strings Number lines Everyday objects
3	Recall and use multiplication and division facts for the 4-times table	Hundred squares Numicon Counters	Bead strings Number lines Everyday objects
3	Recall and use multiplication and division facts for the 8-times table	Hundred squares Numicon	Bead strings Number tracks Everyday objects
4	Recall and use multiplication and division facts for the 6-times table	Hundred squares Numicon	Bead strings Number tracks Everyday objects
4	Recall and use multiplication and division facts for the 7-times table	Hundred squares Base 10	Place value counters Number lines
4	Recall and use multiplication and division facts for the 9-times table	Hundred squares Base 10	Place value counters Number lines
4	Recall and use multiplication and division facts for the 11-times table	Hundred squares Base 10	Place value counters Number lines
4	Recall and use multiplication and division facts for the 12-times table	Hundred squares Base 10	Place value counters Number lines

### Skill: 2 times table

Year: 2

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

Use different models to develop fluency.

### Skill: 5 times table

Year: 2

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

### Skill: 10 times table

Year: 2

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits—the ones are always 0, and the tens increase by 1 ten each time.

### Skill: 3 times table

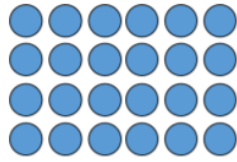
Year: 3

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

### Skill: 4 times table

Year: 3

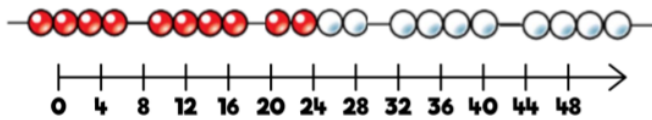


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



4    8    12    16

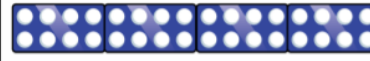
4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the two. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

### Skill: 8 times table

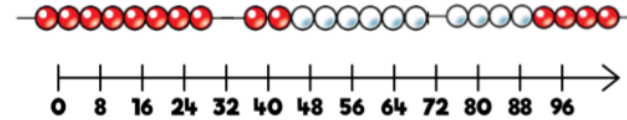
Year: 3



8    16    24    32

8	16	24	32	40
48	56	64	72	80

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the four. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

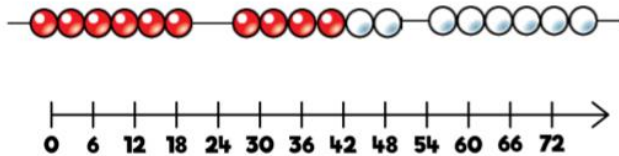
### Skill: 6 times table

Year: 4



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6	12	18	24	30
36	42	48	54	60
66	72	78	84	90



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the three. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

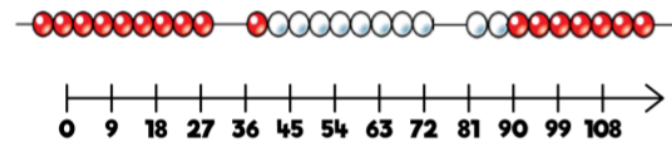
### Skill: 9 times table

Year: 4



9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.



### Skill: 7 times table

Year: 4



7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.



0	7	14	21	28	35	42	49	56	63	70	77	84
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### Skill: 11 times table

Year: 4

11	22	33	44	55	66
77	88	99	110	121	132



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100



0	11	22	33	44	55	66	77	88	99	110	121	132
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### Skill: 12 times table

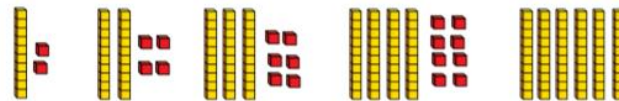
Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			



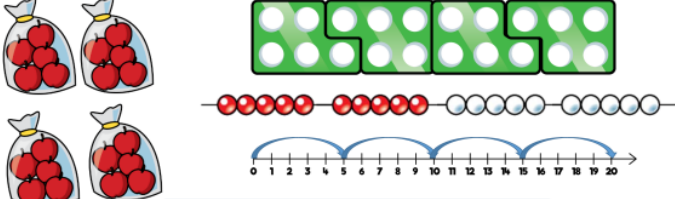
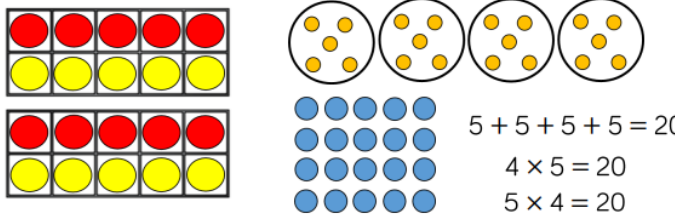
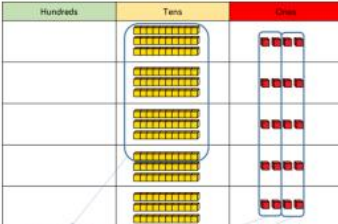
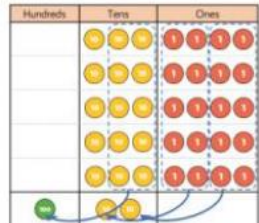
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.



0	12	24	36	48	60	72	84	96	108	120	132	144
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# Representations and Models – Multiplication

Typical Year Group	Skill	Representations and Models																																																	
1/2	Solve one-step problems with multiplication	Bar model Numicon Counters	Tens frames Bead strings Number lines   <p>One bag holds 5 apples. How many apples do 4 bags hold?</p>  $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$																																																
3/4	Multiply 2-digit by 1-digit numbers	Place value counters Base 10	Short written method Expanded written method   <table border="1" data-bbox="1848 813 2072 1037"> <thead> <tr> <th></th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>3</td> <td>4</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>5</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>0</td> </tr> <tr> <td></td> <td>1</td> <td>5</td> <td>0</td> </tr> <tr> <td></td> <td>1</td> <td>7</td> <td>0</td> </tr> </tbody> </table> <p><b><math>34 \times 5 = 170</math></b></p>  <table border="1" data-bbox="1467 1133 1713 1356"> <thead> <tr> <th></th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>3</td> <td>4</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>5</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>0</td> </tr> <tr> <td></td> <td>1</td> <td>7</td> <td>0</td> </tr> <tr> <td></td> <td>1</td> <td>2</td> <td></td> </tr> </tbody> </table>		H	T	O			3	4	x			5			2	0		1	5	0		1	7	0		H	T	O			3	4	x			5			2	0		1	7	0		1	2	
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	1	2																																																	

4

Multiply 3-digit by 1-digit numbers

Place value counters  
Base 10

Short written method

	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	

**$245 \times 4 = 980$**

### Conceptual variation; different ways to ask children to solve $6 \times 23$

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.  
How many lengths did she swim in one week?

With the counters, prove that  $6 \times 23 = 138$

Find the product of 6 and 23

$6 \times 23 =$

   =  $6 \times 23$

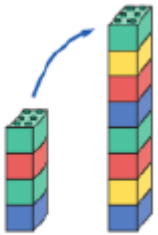
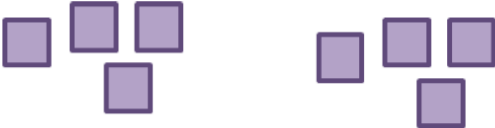
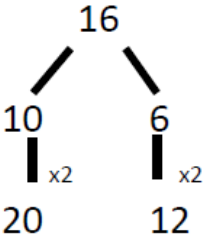
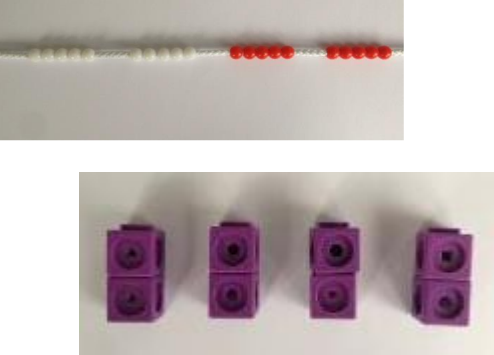
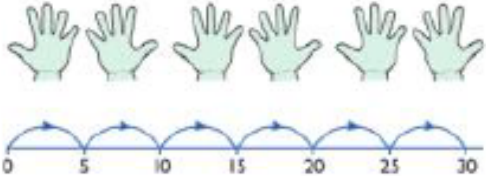
6	23
x 23	x 6
—	—

What is the calculation?  
What is the product?

100s	10s	1s																																										
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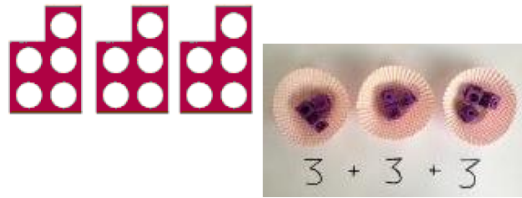
## Progression in calculations: Multiplication

Key Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups

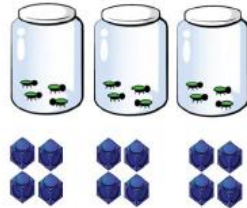
Objective and Strategies	Concrete	Pictorial	Abstract
<p><b>Doubling</b></p>	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
<p><b>Counting in multiples (e.g 2s/5s/10s)</b></p>	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

## Repeated addition

Use different objects to add equal groups practically.



There are 3 equal groups, with 4 in each group.



Using a number line to show repeated groups -

$$3 \times 4$$



Cuisenaire rods can be used too.

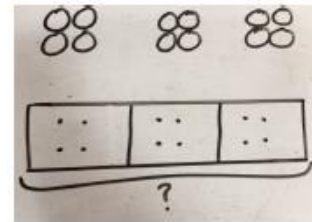
Draw pictures to represent multiplication/repeated addition -

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

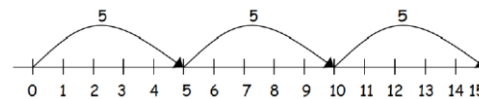
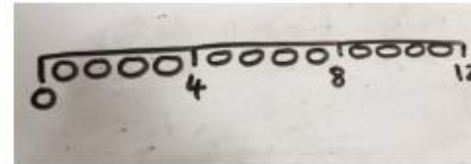


$$2 \text{ add } 2 \text{ add } 2 \text{ equals } 6$$

Children to represent the practical resources in a picture and use a bar model.



Pictorial representation of repeated addition on a number line



$$5 + 5 + 5 = 15$$

Write addition sentences to describe objects and pictures.



$$2 + 2 + 2 + 2 + 2 = 10$$

$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

Abstract number line showing three jumps of 4.

$$3 \times 4 = 12$$

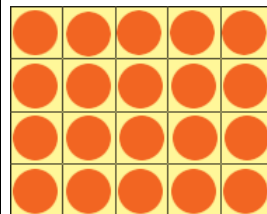


## Arrays showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.

$$4 \times 2 = 8$$

$$2 \times 4 = 8$$

$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

Use an array to write multiplication sentences and reinforce repeated addition.



$2 \times 5$

$5 \times 2$



$5 \times 3 = 15$

$3 \times 5 = 15$

$5 + 5 + 5 = 15$

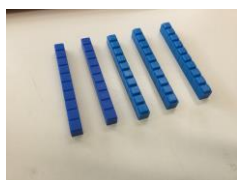
$3 + 3 + 3 + 3 + 3 = 15$

## Multiplying by 10

Time must be taken to explicitly teach and ensure that children understand multiplying by 10 as 10 times or 'lots of' the number (linked to place value) rather than simply applying the rule 'just add a zero'.

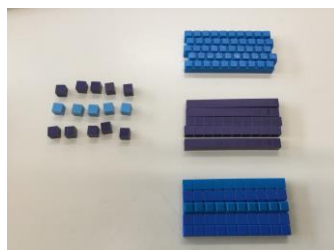
Use Base 10 materials to represent the number sentence

e.g  
 $5 \times 10 =$

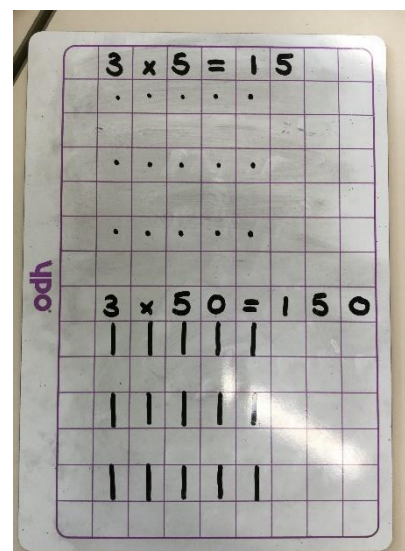


$3 \times 5 =$

$3 \times 50 =$



Remember, children should work with representations of 10 in both horizontal and vertical orientations.



Repeated addition number sentence

$5 + 5 + 5 = 15$

$50 + 50 + 50 = 150$

Into

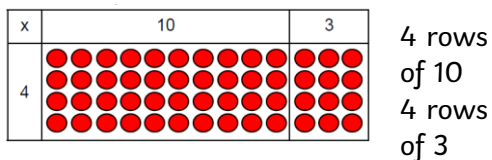
Multiplication number sentence

$3 \times 5 =$

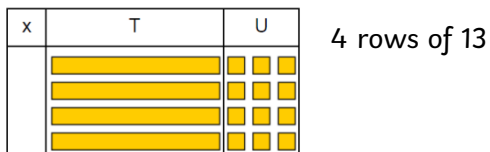
$3 \times 50 =$

# Grid Method

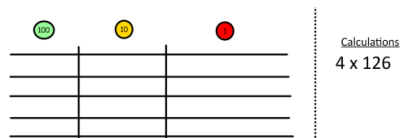
Use partitioning to multiply (hundreds) tens and ones. Show the link with arrays to first introduce the grid method.



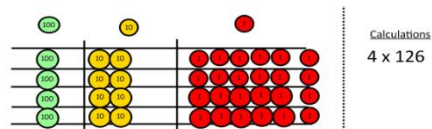
Move on to using Base 10 to move towards a more compact method.



Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.

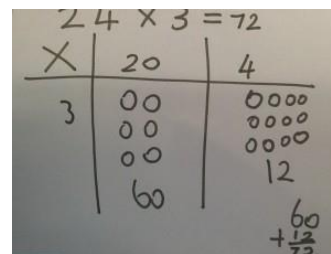
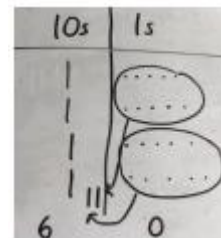


Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with Base 10 / place value counters in a way that they understand.

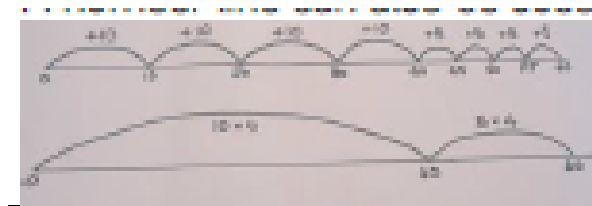
15 x 4 =



They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

A number line can also be used to multiply by partitioning.

4 x 15 =



Expanded grid method recording -  
15x4 =

10 x 4 = 40  
5 x 4 = 20  
40+20 = 60

Into compact grid method

X	10	5
4	40	20

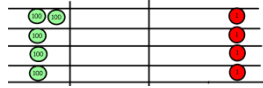
40 + 20 = 60

X	30	5
7	210	35

210 + 35 = 245

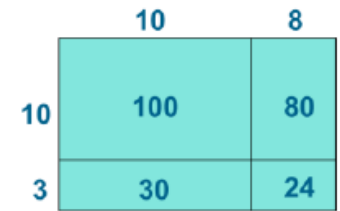
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.



Then you have your answer.

$13 \times 18 =$



$24 \times 32 =$

	30	2
20	600	40
4	120	8
	720	48

$18 \times 1342 =$

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

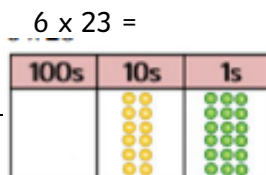
## Column method multiplication

Children can continue to be supported by place value counters at the stage of multiplication.

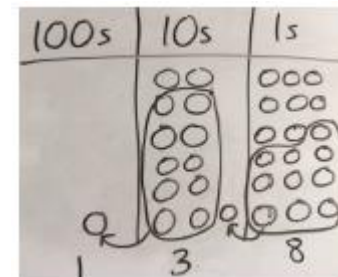
$3 \times 23 =$



It is important at



Children represent calculation using place value counters;



Expanded column method first - to secure place value understanding.

If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r} 30 \ 5 \\ \times \quad 7 \\ \hline 35 \ (7 \times 5) \end{array}$$



this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

210 (7x30)

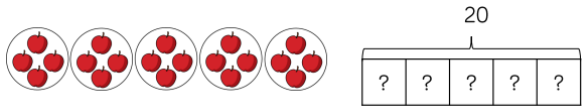
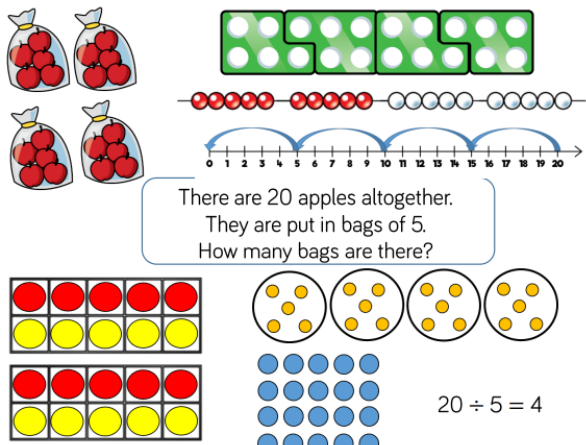
$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 \underline{600} \quad (20 \times 30) \\
 768
 \end{array}$$

This moves to the more compact method.

$$\begin{array}{r}
 74 \\
 \times 63 \\
 \hline
 222 \\
 4440 \\
 \hline
 4662
 \end{array}$$

$$\begin{array}{r}
 6 \times 23 = \\
 \quad \quad \quad 23 \\
 \quad \quad \times 6 \\
 \hline
 \quad \quad 138 \\
 \quad \quad 11 \\
 \hline
 \quad \quad 2 \quad 3 \quad 1 \\
 \quad \quad 1342 \\
 \times \quad 18 \\
 \hline
 13420 \\
 10736 \\
 \hline
 24156
 \end{array}$$

# Representations and Models – Division

Typical Year Group	Skill	Representations and Models	
1/2	Show one-step problems with division (sharing)	<p>Bar model Real life objects</p>	 <p>There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?</p> <p><math>20 \div 5 = 4</math></p>
1/2	Show one-step problems with division (grouping)	<p>Real life objects Numicon Bead strings Ten frames</p>	 <p>There are 20 apples altogether. They are put in bags of 5. How many bags are there?</p> <p><math>20 \div 5 = 4</math></p>

3

Divide 2-digits by 1-digit (no exchange)

Straws  
Base 10  
Bar model

Place value counters  
Part-whole model

Visual models for  $48 \div 2 = 24$ . At the top left is a place value chart with 'Tens' and 'Ones' columns. The 'Tens' column has two rows of two yellow tens blocks each. The 'Ones' column has two rows of four red ones blocks each. To the right are two groups of straws, each containing two bundles of ten and eight individual straws. Below the chart is a part-whole model: a circle labeled '48' branches into two circles labeled '40' and '8'. Arrows labeled  $\div 2$  point from '40' and '8' to empty boxes. To the right is a blue box containing the equation  $48 \div 2 = 24$ . Further right are two circular diagrams, each showing two yellow tens rods and four red ones units.

3/4

Divide 2-digits by 1-digit (sharing with exchange)

Straws  
Base 10  
Bar model

Place value counters  
Part-whole model

Visual models for  $52 \div 4 = 13$ . At the top left is a place value chart with 'Tens' and 'Ones' columns. The 'Tens' column has four rows of one yellow ten block each. The 'Ones' column has four rows of two red ones blocks each. To the right is a bar model for 52 divided by 4, with a box containing '52' above a bar divided into four equal sections, each containing a question mark. Below the chart is a part-whole model: a circle labeled '52' branches into two circles labeled '40' and '12'. Arrows labeled  $\div 4$  point from '40' to '10' and from '12' to '3'. Below these are the equations  $10 + 3 = 13$ . To the right is a blue box containing the equation  $52 \div 4 = 13$ . Further right is a place value chart with 'Tens' and 'Ones' columns. The 'Tens' column has one yellow ten block in the first row and three yellow ten blocks in the second, third, and fourth rows. The 'Ones' column has one row of three red ones blocks and three rows of one red one block each.

3/4

Divide 2-digits by 1-digit (sharing with remainders)

Straws  
Base 10  
Bar model

Place value counters  
Part-whole model

$53 \div 4 = 13 \text{ r}1$

4/5

Divide 2-digits by 1-digit (grouping)

Base 10  
Bar model

Place value counters  
Part-whole model

$52 \div 4 = 13$

4

Divide 3-digits by 1-digit (sharing with exchange)

Place value counters  
Counters

Place value grid  
Written short division

$844 \div 4 = 211$

## Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

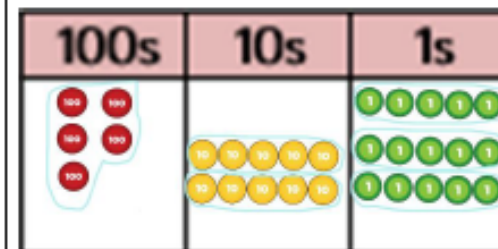
615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

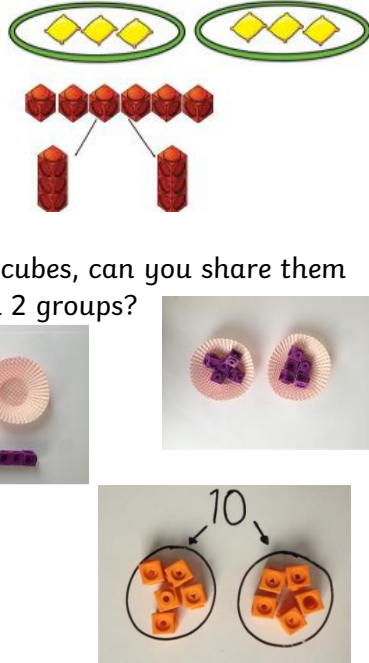
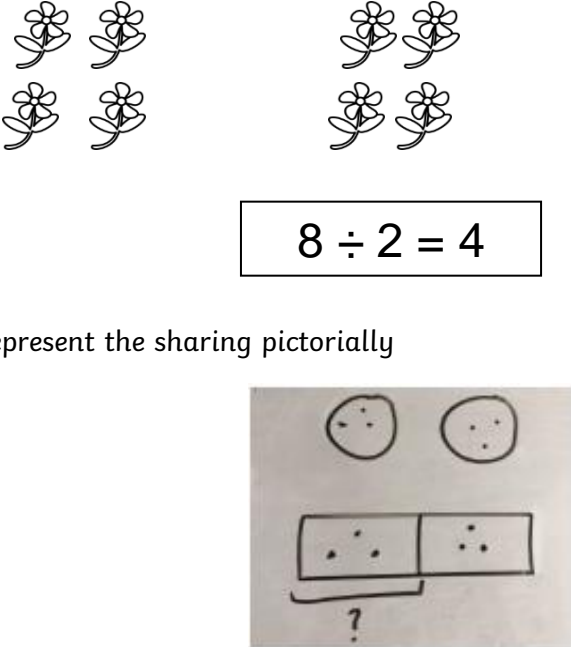
$$\square = 615 \div 5$$

What is the calculation?  
What is the answer?



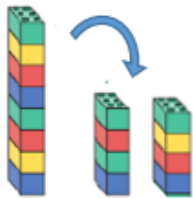
# Progression in calculations: Division

Key Vocabulary: share, group, divide, divided by, half.

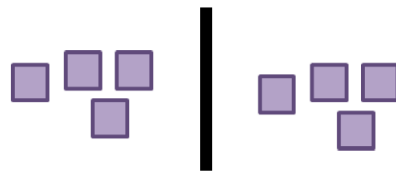
Objective and Strategies	Concrete	Pictorial	Abstract		
<p style="text-align: center;"><b>Sharing objects into groups</b></p>	<p>Exploring sharing into equal groups using a range of objects.</p>  <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p style="text-align: center;"><math>8 \div 2 = 4</math></p> <p>Represent the sharing pictorially</p>	<p>Share 9 buns between three people.</p> <p style="text-align: center;"><math>9 \div 3 = 3</math></p> <p><math>6 \div 2 = 3</math></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">3</td> </tr> </table> <p>Children should be encouraged to use known number facts and knowledge of inverse.</p>	3	3
3	3				

## Find half – sharing or dividing by two

Use practical activities to show how to halve a number by splitting a group into 2 equal parts .



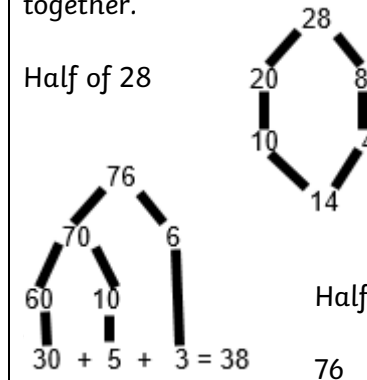
Draw pictures to show how to halve a number.



Half of 8 is 4

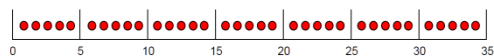
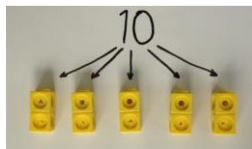
Partition a number and then half each part before recombining it back together.

Half of 28



## Division as grouping (repeated subtraction)

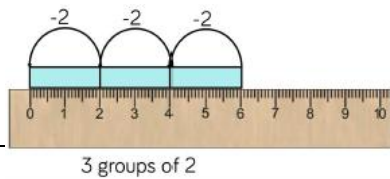
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.



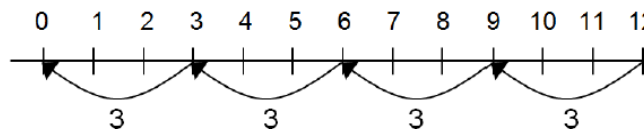
$$96 \div 3 = 32$$



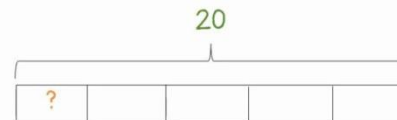
Cuisenaire rods above a ruler  $6 \div 2 = 3$



Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



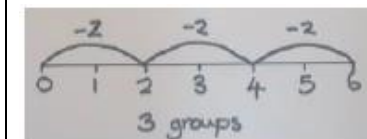
$$20 \div 5 = ?$$

$$5 \times ? = 20$$

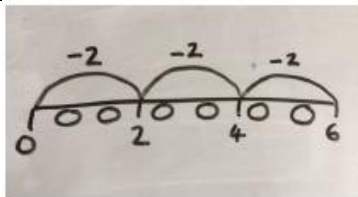
Children represent repeated subtraction pictorially

$$28 \div 7 = 4$$

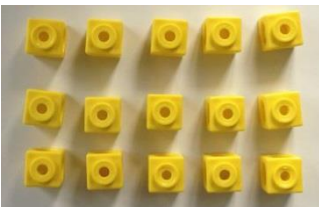
Divide 28 into 7 groups. How many are in each group?



Abstract number line to represent the groups that have been subtracted.



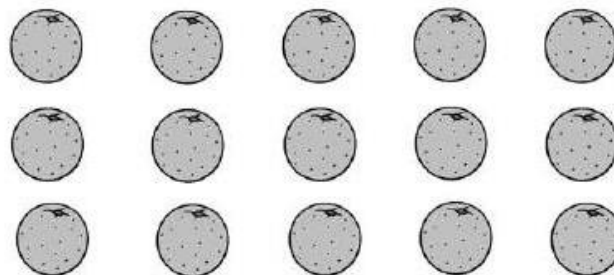
### Division within arrays (inverse)



Link division to

multiplication by creating an array and thinking about the number sentences that can be created.

Eg  $15 \div 3 = 5$      $5 \times 3 = 15$   
 $15 \div 5 = 3$      $3 \times 5 = 15$



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

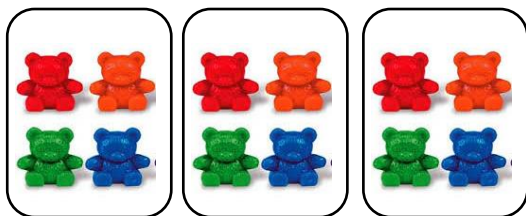
Find the inverse of multiplication and division sentences by creating four linking number sentences.

$7 \times 4 = 28$   
 $4 \times 7 = 28$   
 $28 \div 7 = 4$   
 $28 \div 4 = 7$

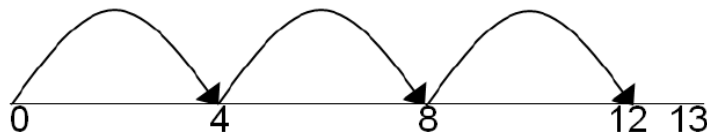
### Division with a remainder

$14 \div 3 =$

Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



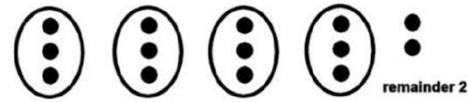
Complete written divisions and show the remainder using r.

$29 \div 8 = 3 \text{ REMAINDER } 5$   
 ↑    ↑    ↑                    ↑  
 dividend    divisor    quotient                    remainder





Draw dots and group them to divide an amount and clearly show a remainder.



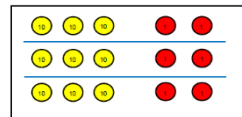
### Sharing with place value counters.

Use place value counters to divide using the bus stop method alongside

$$96 \div 3 = 32$$

Tens	Units
3	2

3

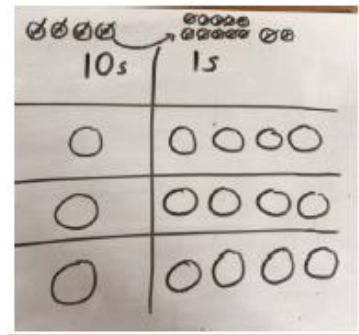


$42 \div 3 =$   
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



10s	1s

Children to represent the place value counters pictorially



Children make sense of the place value counters and write calculations to show the process

$$42 \div 3 =$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

10s	1s
●	
●	
●	

We exchange this ten for ten ones and then share the ones equally among the groups.

10s	1s
●	●●●●●●●●
●	●●●●●●●●
●	●●●●●●●●

10s	1s
●	●●●●
●	●●●●
●	●●●●

We look how much in 1 group so the answer is 14.

Short division using place value counters to group.

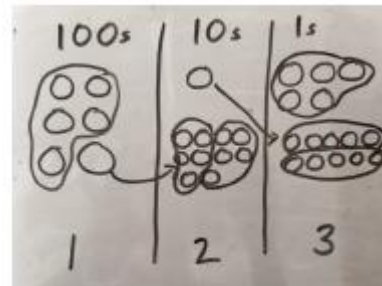
$$615 \div 5 =$$

123

100s	10s	1s
●●●	●●●●●●	●●●●●●●●●●
●●●	●●●●●●	●●●●●●●●●●
●●●	●●●●●●	●●●●●●●●●●

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 tens counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially



$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \phantom{0} \\ 11 \phantom{0} \\ \underline{10} \phantom{0} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \phantom{0} \\ 7 \phantom{0} \\ \underline{8} \\ 2 \end{array}$$

$$\begin{array}{r} 87 \\ 5 \overline{) 435} \\ \underline{4} \phantom{0} \\ 3 \phantom{0} \\ \underline{3} \\ 5 \end{array}$$

## Short division

Can be partitioned to secure understanding of place value and how many groups of \* can be made.

$$\begin{array}{r} 1 \quad 2 \quad 3 \\ 5 \overline{) 600 \quad 10 \quad 5} \end{array}$$

$$\begin{array}{r} 2 \quad 1 \quad 8 \\ 4 \overline{) 800 \quad 70 \quad 2} \end{array}$$

$$\begin{array}{r} 8 \quad 7 \\ 5 \overline{) 400 \quad 30 \quad 5} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 8 \quad 6 \quad r \quad 2 \\ 3 \\ 5 \overline{) 4 \quad 3 \quad 2} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 1 \quad 4 \quad . \quad 6 \\ 16 \quad 21 \\ 3 \quad 5 \overline{) 5 \quad 1 \quad 1 \quad . \quad 0} \end{array}$$

## Progression in addition and subtraction of fractions and decimals

Year 3	Year 4	Year 5
<p><u>Fractions</u></p> <p>Addition of fractions with the same denominator within a whole number.</p> <p>e.g <math>5/7 + 1/7 = 6/7</math> (answer will be less than a whole.)</p>	<p><u>Fractions</u></p> <p>Addition of 2 fractions totalling more than a whole written as a mixed number.</p> <p>e.g. <math>2/3 + 2/3 = 1\ 1/3</math> (not <math>4/3</math>)</p> <p><u>Decimals</u></p> <p>Dividing 1 or 2 digit numbers by 10 or 100 up to 2 decimal places (this fits in well with money) Recap multiplying by 10 before dividing by 10. Ensure children move the numbers/digits, not just moving the decimal point.</p>	<p><u>Fractions</u></p> <p>Addition and subtraction of fractions with the same denominator and multiples of the same number.</p> <p><u>Decimals</u></p> <p>Addition of numbers with up to 3 decimal places.</p> <p>2.32 + 4.71</p> <p>Further extension and challenge – add 2 decimal places to 3 decimal places – understanding of place value must be secure to ensure numbers are lined up accurately.</p> <p>2.32 + 4.718</p>